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Adding momentum to D1-D5 system

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Abstract: We construct the first example of asymptotically flat solution which carries three charges (D1, D5 and momentum) and which is completely regular everywhere. The construction utilizes the relation between gravity solutions and spectral flow in the dual CFT. We show that the solution has the right properties to describe one of the microscopic states which are responsible for the entropy of the black hole with three charges.

Keywords: Black Holes in String Theory, AdS-CFT and dS-CFT Correspondence.

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1. Introduction

One of the most fascinating problems in theoretical physics is the problem of black hole entropy. Ever since Bekenstein first proposed his famous formula [1] relating the entropy of black hole with the area of the horizon, there were countless attempts to identify the microscopic states responsible for such entropy. A big step towards understanding of microscopic states was made in [2]. For a specific type of black holes which arises in string theory and which has three charges (corresponding to D1 branes, D5 branes and momentum), Strominger and Vafa counted the microscopic states of the branes and demonstrated perfect agreement with Bekenstein formula. This counting was later extended to the rotating [3] and near extremal [4] black holes.

While the work of Strominger and Vafa [2] explained how to *count* the states of the black hole, it did not address the question how to *see* them as different states in supergravity. Unfortunately the gravity picture for the microscopic states is still missing.

It is interesting that for a somewhat simpler system which has only two charges (D1 and D5, but no momentum), one can actually construct the geometries corresponding to *all* microscopic states [5, 6]. This system does not form a black hole, but it still has many degenerate microscopic states ($\sim \exp(2\sqrt{2}\pi\sqrt{n_1 n_5})$ of them). In [5] a large class of these microscopic states was considered and the geometries corresponding to each of those states were constructed.[1] In [6] this procedure was repeated for the remaining states,[2] and more importantly, it was proven that geometries corresponding to all microscopic states are completely regular. As we mentioned, D1-D5 system does not have a horizon, but it does have degenerate states and thus it does have entropy. The way to explain an entropy for systems without horizons was proposed by Sen [8] and it was based on a concept of a stretched horizon. It is interesting that for D1-D5 system one can see the emergence of an effective stretched horizon from the geometries which are completely regular, and the area of this stretched horizon reproduces the correct expression for the entropy [9].

Ideally one wants to have the same understanding of the three charged system as well. The first steps in this direction were made recently in [10, 11], where some excitations of the D1-D5 systems were considered. These excitations corresponded to putting a quantum with nonzero momentum on a D1-D5 system, thus producing the system with two large charges and one small charge. Unfortunately for the ansatz taken in [10] the equations were too complicated to get a solution even on the linearized level, but the authors of [10] made an extensive use of matching techniques to show convincingly that the solution of such linearized equations should exist and it should be regular.

In a seemingly unrelated line of development, recently there was a significant progress in understanding the properties of supergravity solutions in various dimensions with various amounts of supersymmetry [12, 13]. In particular, in [12] a classification of all supersymmetric solutions of minimal six dimensional supergravity was performed, and as we will see these are the solutions which are relevant for understanding of the microstates of three charged black hole. To be more precise, the classification of [12] did not give an explicit form of a general supersymmetric solution, and the examples considered in [12] had horizons and singularities, but [12] derived a set of (nonlinear partial differential) equations which should be satisfied by all supersymmetric solutions. While we are not aware of any method of solving such PDEs, we will see that intuition from D1-D5 system can be used to construct a particular solution which has three charges and which is completely regular.

Our construction is inspired by the work of [14, 15], where the first and the simplest example of regular geometry for D1-D5 system was presented. Let us briefly review their arguments. According to AdS/CFT correspondence [16, 17], near horizon limit of D1-D5 system is dual to a state in two dimensional CFT. However if we start from asymptotically flat geometry, fermions are necessarily periodic as we go around the spacial direction in CFT (i.e. they belong to the Ramond sector). On the other hand, the vacuum of CFT belongs to NS sector, so it has antiperiodic fermions. In two dimensional conformal field theory one usually goes between R and NS sector by a procedure known as spectral flow. In [14, 15] it was shown that the spectral flow in CFT corresponds to a diffeomorphism on the gravity side. Moreover, by applying a spectral flow to the NS vacuum, [14, 15] produced a geometry which represented a near horizon limit of the black hole of [18], and

[1] For earlier work on geometries of the two charge system see for example [7].

[2] The five dimensional black hole is constructed by compartifying string theory on either T^4 or K3. The work [5] dealt with microstates which are universal and appear for both compactifications. In [6] the states associated with excitations on T^4 were constructed explicitly, and the procedure for constructing excitations associated with K3 was outlined.

thus it could be continued all the way to flat infinity. The CFT interpretation of the near horizon limit led to a particular relation between the parameters of the solution, and for this set of parameters [14, 15] demonstrated that the solution was regular. While we are not planning to discuss the general relation between spectral flow and gravity solutions (we just refer to [14, 15, 19]), we will apply an additional spectral flow to the solution of [14, 15]. This would allow us to construct a regular solution with three charges. It is less trivial than it sounds because the spectral flow would only give us solution in the near horizon region, and then we will have to use the technology of [12] to find the interpolating functions which make the solution asymptotically flat rather than $AdS_3 \times S^3$.

In order to describe a microscopic state contributing to the entropy of D1-D5-P black hole, the geometry should satisfy the following requirements:[3]

1. The solution is asymptotically flat.
2. The solution does not have curvature singularities.
3. The solution does not have a horizon.
4. The solution has three charges.
5. The solution is supersymmetric.

We will demonstrate that our solution possesses all these properties, and thus it is a good candidate for a microscopic state of a three charged black hole.[4]

The paper has the following structure. In section 2 we review a construction of [12] and we show that the regular solutions of [5] can be embedded in this construction when two formalisms overlap. In section 3 we introduce a map which relates two solutions of equations coming from [12]: a solution with flat asymptotics is related to a solution which asymptotes to $AdS_3 \times S^3$. We call this a "near horizon map" and we show that it is invertible. In section 4 we use the spectral flow and the near horizon map in order to construct an asymptotically flat space with three charges. Section 5 is devoted to the analysis of that solution, in particular we show that it satisfies the requirements 1–5 imposed on the microscopic state. Finally we discuss the relation of our geometry to other approaches to the three charged system, and we also make some comments about possible applications to AdS/CFT.

2. Six dimensional supergravity and D1-D5 system

In this section we will briefly summarize some of the results of [12] which will be useful for our discussion.

In [12], Gutowski, Martelli and Reall studied supersymmetric solutions of six dimensional supergravity, and remarkably they found the most general form of such solutions.

[3] The most nontrivial requirements are 2 and 3 and they were spelled out in [10].

[4] We should mention however that for generic values of parameters the angular momenta of our solution are not the same as the angular momenta of BMPV black hole [3], so one should not view our state as a *typical* state contributing to the entropy of the black hole.

The theory under consideration consisted of graviton, two-form field $B_{\mu\nu}$ and a symplectic Majorana-Weyl gravitino ψ_μ^A. The equations of motion for the bosonic fields are [20, 12]:

$$\begin{aligned} G &= dB\,, \qquad G =\star_6 G\,, \\ R_{\mu\nu} &= G_{\mu\rho\sigma}G_\nu{}^{\rho\sigma}\,. \end{aligned} \tag{2.1}$$

The authors of [12] were interested in constructing solutions which had at least one Killing spinor satisfying

$$\nabla_\mu\epsilon - \frac{1}{4}G_{\mu\nu\lambda}\gamma^{\nu\lambda}\epsilon = 0\,. \tag{2.2}$$

Such solutions would have at least four supersymmetries. By studying restrictions which are imposed by the mere existence of the Killing spinor, GMR were able to show that the metric of the supersymmetric solution can always be written in the form:[5]

$$ds^2 = -2H^{-1}(du+\beta_m dx^m)\left(dv+\omega_m dx^m + \frac{F}{2}(du+\beta_m dx^m)\right) + Hh_{mn}dx^m dx^m\,. \tag{2.3}$$

Here h_{mn} is a hyper-Kähler metric of the base space. We assume that this metric as well as all other functions entering the ansatz are functions of x_m only.[6] Under this assumption the equations of [12] simplify considerably, and we summarize them here:

$$\begin{aligned} d\beta &= {}^\star d\beta \\ d^\star dH + d\beta\wedge\mathcal{G}^+ &= 0 \\ d\mathcal{G}^+ &= 0 \\ {}^\star d^\star dF &= (\mathcal{G}^+)_{mn}(\mathcal{G}^+)^{mn},\,. \end{aligned} \tag{2.4}$$

Here and below an expression ${}^\star A$ means taking the Hodge dual with respect to *four dimensional base space* with metric h_{mn}, and the superscript $\pm$ denotes (anti)self-dual part of a two-form under this Hodge duality. We also introduce the following notation:

$$\mathcal{G}^+ \equiv H^{-1}((d\omega)^+ + \frac{1}{2}Fd\beta)\,. \tag{2.5}$$

For completeness we also give an expression for the tree-form field strength [12]:

$$\begin{aligned} G = &\frac{1}{2}\,{}^\star dH - \frac{1}{2}H^{-1}(du+\beta)\wedge(d\omega)^- + \\ &+\frac{1}{2}H^{-1}\left[dv+\omega+\frac{F}{2}(du+\beta)\right]\wedge[d\beta+(du+\beta)\wedge dH]\,. \end{aligned} \tag{2.6}$$

In [12] it was shown that any solution of the system (2.4)–(2.6) gives a supersymmetric solution of the minimal six dimensional supergravity. These solutions can be easily embedded in type IIB supergravity [21] by treating $B_{\mu\nu}$ as NS-NS (or R-R) two form and treating the metric (2.4) as a six-dimensional part of the string metric:

$$ds_{10}^2 = ds_6^2 + dz_1^2 + dz_2^2 + dz_3^2 + dz_4^2\,. \tag{2.7}$$

[5]Here and below we are slightly modifying the notation of [12] to make it simpler for the applications which we have in mind, but one can easily trace our notation to the original notation of [12]

[6]Generically h_{mn}, β_m, ω_m, H and F can also depend upon the coordinate u, see [12] for details.

Since neither dilaton, nor moduli controlling the size of four dimensional torus are excited, we don't have to distinguish between string and Einstein frames. Depending on the type of the two form (NS-NS or R-R) the configuration describes either a system of fundamental strings and NS5 branes, or a system of D1 and D5 branes. For concreteness we will always be talking about D1-D5 system.

Unfortunately, most of the solutions of (2.4)–(2.5) have curvature singularities, but there exists a large class of the solutions which are completely regular. These solutions were first constructed in [5], and their smoothness was proven in [6]. While the regular solutions of [5] generically lie outside the scope of minimal supergravity (for example, they have nontrivial dilaton), there is an overlap between solutions of [12] and regular solutions of [5]. Let us briefly summarize the properties of this subclass.

In [5] the solutions for D1-D5 system were parameterized in terms of a four dimensional vector $\mathbf{F}(\xi)$ (which was a function of one variable ξ) and a single charge Q. As we mentioned, generically these solutions do not reduce to the solutions of minimal six dimensional supergravity, but they do reduce to the solutions of [12] if the vector satisfies the condition:

$$\dot{\mathbf{F}}^2(\xi) = 0\,. \tag{2.8}$$

Using this profile one constructs a harmonic function H and a gauge field A_i:

$$H = 1 + \frac{Q}{L}\int_0^L \frac{d\xi}{[\mathbf{x} - \mathbf{F}(\xi)]^2}\,, \qquad A_i = -\frac{Q}{L}\int_0^L \frac{\dot{F}_i(\xi) d\xi}{[\mathbf{x} - \mathbf{F}(\xi)]^2}\,, \tag{2.9}$$

and constructs the field B_i dual to A_i with respect to a *flat* four dimensional metric:

$$dB = -^\star\, dA\,. \tag{2.10}$$

The regular solution of the D1-D5 system can then be constructed in terms of these data:

$$\begin{aligned} ds^2 &= H^{-1}\left[-(dt - A_m dx^m)^2 + (dy + B_m dx^m)^2\right] + H\delta_{mn} dx^m dx^n + d\mathbf{z}d\mathbf{z} \\ G^{(3)} &= d\left\{H^{-1}(dt - A_m dx^m) \wedge (dy + B_m dx^m)\right\} -^\star dH \end{aligned} \tag{2.11}$$

To compare this with GMR solution (2.4), (2.6), we introduce the light-like coordinates u, v as well as self-dual and anti-self-dual fields a_m, b_m:

$$u = \frac{t+y}{\sqrt{2}}, \quad v = \frac{t-y}{\sqrt{2}}\,, \qquad a_m = \frac{B_m - A_m}{\sqrt{2}}\,, \qquad b_m = -\frac{A_m + B_m}{\sqrt{2}}\,. \tag{2.12}$$

Using this notation we can rewrite (2.11) as

$$ds^2 = -2H^{-1}(du + a_m dx^m)(dv + b_m dx^m) + H\delta_{mn} dx^m dx^n + d\mathbf{z}d\mathbf{z} \tag{2.13}$$

$$G^{(3)} = d\left\{H^{-1}(dv + b_m dx^m) \wedge (du + a_m dx^m)\right\} -^\star dH\,. \tag{2.14}$$

We observe the perfect agreement between this solution and (2.4), (2.6) in the region where they overlap,[7] i.e.

$$F = 0\,, \qquad \beta = a_m dx^m\,, \qquad \omega = b_m dx^m \tag{2.15}$$

and thus $(d\omega)^- = d\omega$.

[7] There is an overall factor of -2 between (2.6) and (2.14), which can be traced to different normalization for the two forms used in [5] and [12].

To summarize, so far we have discussed two different sets of solutions of type IIB supergravity reduced to six dimensions. One class [12] gives solutions which have self-dual field strength in six dimensions, and upon lifting to 10d it describes supersymmetric solutions which generically have three charges: D1, D5 and momentum. Unfortunately solutions of this class are not guaranteed to be regular and we do not know any simple way to produce regular solutions in this approach.

Another class is represented by solutions of [5], they all have regular geometries and generically six dimensional field strength is not self-dual and dilaton is excited. Unfortunately, those solutions have only two charges (D1 and D5) since in CFT they correspond to the Ramond vacua

$$L_0 = \bar{L}_0 = \frac{c}{24} \tag{2.16}$$

so the momentum charge is necessarily zero. We also saw that in the region where both formalisms can be applied (i.e. for solutions without dilaton and without momentum charge) they give the same results.

In the next section we will try to combine the virtues of two approaches: microscopic intuition (which was ultimately responsible for regularity of the solutions in [5]) and the power of supersymmetry (which was responsible for the completely general ansatz of [12]) to construct an example of a regular solution which has three charges (D1, D5 and momentum).

3. Regular solutions: far away and up-close

We are interested in getting solutions of (2.4)–(2.6) which are regular and which have flat asymptotics. Specifically we will require that the base metric h_{mn} is asymptotically flat, the function F as well as one forms β, ω die off at infinity, while H can be represented as

$$H = 1 + \hat{H}\,, \tag{3.1}$$

where $\hat{H}$ goes to zero at infinity. This representation of H is particularly useful for taking the near horizon limit [16].

Let us start from the D1-D5 solutions (2.11). Then one goes to the near horizon region by replacing H by $\hat{H}$. Since equations (2.5) in this case reduce to

$$d\beta =^\star d\beta, \qquad d^\star dH = 0, \qquad d\omega = -^\star d\omega \tag{3.2}$$

it is clear that the near horizon region defined in such way is a solution of the same equations as the original system. Of course, the near horizon solution is no longer asymptotically flat, it asymptotes to $AdS_3 \times S^3$, and this fact will be important in the next section.

Let us now try to define an analog of the near horizon region for the generic solution of (2.4)–(2.5). We still want to have $H \to \hat{H}$ in this region, but now this would not be enough because this replacement modifies the expression for $\mathcal{G}^+$ and the equations (2.5) will no longer be satisfied. The simplest way to resolve this problem is to modify ω as well in such a way that $\mathcal{G}^+$ does not change. More explicitly, we will define the "near horizon region" as a solution obtained by a replacement

$$H = 1 + \hat{H} \to \hat{H}, \qquad \omega \to \hat{\omega} \tag{3.3}$$

such that

$$H^{-1}\left((d\omega)^+ + \frac{1}{2}F d\beta\right) = \hat{H}^{-1}\left((d\hat{\omega})^+ + \frac{1}{2}F d\beta\right). \tag{3.4}$$

In other words,

$$\hat{H} = H - 1\,, \qquad (d\hat{\omega})^+ = \frac{H-1}{H}(d\omega)^+ - \frac{1}{2}\frac{F}{H}d\beta\,, \tag{3.5}$$

Notice that so far the "near horizon region" was just a name for the procedure which maps one solution of (2.4)–(2.5) into another one (but with different asymptotics). Going to this region was not associated with taking any kind of a limit, this fact allows us to invert the map:

$$H = \hat{H} + 1\,, \qquad (d\omega)^+ = \frac{\hat{H}+1}{\hat{H}}(d\hat{\omega})^+ + \frac{1}{2}\frac{F}{\hat{H}}d\beta\,. \tag{3.6}$$

We arrived at the main observation of this paper. Given an asymptotically flat solution parameterized by $(H, F, \beta, \omega, h_{mn})$ we can construct a solution with $AdS_3 \times S^3$ asymptotics which is parameterized by $(\hat{H}, F, \beta, \hat{\omega}, h_{mn})$ by applying the map (3.5). And vice versa, starting from solution which asymptotes to $AdS_3 \times S^3$, we can construct an asymptotically flat solution using the map (3.6). These maps give a one-to-one correspondence between solutions with flat and $AdS_3 \times S^3$ asymptotics if we also require that ω decays at infinity.

Notice that the term "near horizon region" was inspired by the term "near horizon limit" used for the black holes [16]. Of course, in our case we hope to avoid horizons altogether (and we will show that this indeed happens for the explicit solution), but it is still convenient to give some special name to the solution $(\hat{H}, F, \beta, \hat{\omega}, h_{mn})$ and we choose this name to be "near horizon region." We hope that this would not lead to confusion and we want to stress again that the term has nothing to do with horizon. It is also interesting that the map (3.5) can sometimes be understood as a limit which is analogous to the near horizon limit for the black holes and we discuss this limit in more detail in the appendix A.

Now we can proceed in constructing the solution for D1-D5 system with momentum charge. We will use the following strategy. First we take one of the regular solutions from the family (2.11) and go to the near horizon region using (3.5). Then we apply a diffeomorphism in $AdS_3 \times S^3$ to introduce a nontrivial function F to the system. Notice that after such diffeomorphism the metric would not generically be in the form (2.4), so the inverse transformation (3.6) cannot be applied.[8] However there is a set of special diffeomorphisms (which are called spectral flow from the CFT point of view) which preserve the structure of GMR ansatz. To be more precise, a generic spectral flow gives a solution of [12] which is u-dependent. The solutions produced by such spectral flows will be analyzed elsewhere, but in the next section we consider the simplest example of the spectral flow

[8]Notice that in [12] is was proven that all supersymmetric solutions *can* be represented in the form (2.4) *after* appropriate change of variables. If after diffeomorphism the solution is not in the form (2.4), this simply means that we chose a bad coordinate system.

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which transforms a static solution of the form (2.4) into another static solution which has the same form. The new solution however has a nontrivial function F, so after applying the inverse transformation (3.6) to it we produce an asymptotically flat solution with nonzero value of the momentum. Since this solution was regular in the "near horizon" region it is plausible that it will be regular everywhere.

Let us summarize our strategy.

1. Start from one of the regular solutions (2.11) and rewrite it in the form (2.13).
2. Go to the "near horizon region" using (3.5).
3. Perform a spectral flow which keeps the solution in the class (2.4), but produces nontrivial F.
4. Change boundary conditions to flat by using (3.6).

In the end we produce a solution which has three charges and which has a good chance to be completely regular. We will present an explicit example of this construction in the next section.

4. An example of a solution with three charges

Let us now implement the general procedure which was outlined in the previous section. We begin with solution describing extremal two charged black hole which was considered in [14, 15].[9] In [14, 15] it was observed that for certain values of the parameters the near horizon region of such black hole becomes $AdS_3 \times S^3$ in *global* coordinates. This regular solution can be viewed as a special case of the general metric (2.11) with a particular profile:[10]

$$F_1(\xi) = a\cos\frac{2\pi\xi}{L}\,, \qquad F_2(\xi) = a\sin\frac{2\pi\xi}{L}\,, \qquad F_3(\xi) = F_4(\xi) = 0\,. \tag{4.1}$$

Notice that while this solution is regular for all values of a and L, generically it falls outside the scope of minimal supergravity (for example, generically it has a nontrivial dilaton). As we discussed before, the solution belongs to minimal supergravity if and only if

$$|\dot{\mathbf{F}}| = 1 \quad \rightarrow \quad L = 2\pi a\,. \tag{4.2}$$

From now on we will assume that this relation is satisfied.

We present the complete solution later (this solution will come out as a special member of a more general class), and here we directly write the near horizon geometry, i.e. we go directly to the second step in our strategy:

$$\begin{aligned} ds^2 =& \frac{1}{Q}\left[-(r^2+a^2)dt^2 + r^2dy^2\right] + Q\frac{dr^2}{r^2+a^2} + \\ &+ Q\left[d\theta^2 + \cos^2\theta(d\psi - \frac{a}{Q}dy)^2 + \sin^2\theta(d\phi - \frac{a}{Q}dt)^2\right]. \end{aligned} \tag{4.3}$$

[9]This solution was a special case of a more general five dimensional black holes constructed in [18].

[10]This way of deriving solution from chiral null model [22] was presented in [23]

Notice that we wrote this metric in a form which is slightly different from (2.3), but which is equivalent to it: one should use the expressions for the null coordinates (2.3) and recombine various terms. The reason we chose this "unconventional" form is that it makes the $AdS_3 \times S^3$ structure explicit and also it makes obvious the fact that metric is regular if the radius[11] of y circle takes a special value [14, 15]:

$$R = \frac{Q}{a}\,. \tag{4.4}$$

Notice that the bosonic metric (4.3) describes $AdS_3 \times S^3$ in global coordinates, but due to nontrivial cross terms between sphere and AdS, the fermions are *periodic* under identification $y \sim y + 2\pi R$, i.e. we are dealing with *Ramond sector* of the corresponding CFT[12]. To see the relation between NS and R sectors more clearly, we rewrite the metric in terms of the coordinates u and v:

$$\begin{aligned} ds^2 &= -\frac{1}{Q}\left[2r^2 du dv + \frac{Q^2}{2R^2}(du+dv)^2\right] + Q\frac{dr^2}{r^2+Q^2/R^2} + \\ &\quad + Q\left[d\theta^2 + \cos^2\theta\left(d\psi - \frac{du}{\sqrt{2}R} + \frac{dv}{\sqrt{2}R}\right)^2 + \sin^2\theta\left(d\phi - \frac{du}{\sqrt{2}R} - \frac{dv}{\sqrt{2}R}\right)^2\right]. \end{aligned} \tag{4.5}$$

The connections on the sphere which appear in the second line of the above expression are responsible for performing the spectral flow from the NS vacuum of CFT. In particular, the connections proportional to du are responsible to the spectral flow in the left sector and the ones proportional to dv correspond to spectral flow in the right sector (see [14, 15, 19] for details). An addition of extra connection proportional to du performs an extra spectral flow in the left sector, while still keeping Ramond vacuum on the right:

$$\bar{L}_0 = \frac{c}{24} \tag{4.6}$$

Let us perform such spectral flow:

$$\begin{aligned} ds^2 &= -\frac{1}{Q}\left[2r^2 du dv + \frac{Q^2}{2R^2}(du+dv)^2\right] + Q\frac{dr^2}{r^2+Q^2/R^2} + Qd\theta^2 + \\ &\quad + Q\left[\cos^2\theta\left(d\psi - (2\nu+1)\frac{du}{\sqrt{2}R} + \frac{dv}{\sqrt{2}R}\right)^2 + \sin^2\theta\left(d\phi - (2\nu+1)\frac{du}{\sqrt{2}R} - \frac{dv}{\sqrt{2}R}\right)^2\right]. \end{aligned} \tag{4.7}$$

In this expression ν is just a parameter which describes the extra spectral flow. However we will be interested in the case where the new geometry is regular, then ν has to be an integer. To see this we observe that the metrics (4.5) and (4.7) are related by diffeomor-

[11] We use identification $y \sim y + 2\pi R$.

[12] This fact is not surprising, since (4.3) was obtained as a near horizon limit of asymptotically flat geometry, where fermions were clearly periodic.

phism:

$$\psi \to \psi - 2\nu\frac{u}{\sqrt{2}R}\,, \qquad \phi \to \phi - 2\nu\frac{u}{\sqrt{2}R}\,, \tag{4.8}$$

and such diffeomorphisms relating periodic variables (we recall that u inherits periodicity from y coordinate: $u \sim u + \sqrt{2}\pi R$) are regular only if ν is integer.[13]

At this point we almost completed step 3 of our program: we found a solution in the near horizon region which has nontrivial F (i.e. it has nontrivial g_{uu}). To be able to extend this solution to the asymptotically flat region, we should recombine it into the form (2.4). Performing a simple algebra, we find the functions parameterizing the solution:

$$\hat{H} = \frac{Q}{r^2+(\nu+1)a^2\cos^2\theta - \nu a^2\sin^2\theta}\,, \quad F = -\frac{2\nu(\nu+1)a^2}{r^2+(\nu+1)a^2\cos^2\theta - \nu a^2\sin^2\theta} \tag{4.9}$$

$$\beta = \frac{aQ}{\sqrt{2}}\frac{\sin^2\theta d\phi - \cos^2\theta d\psi}{r^2+(\nu+1)a^2\cos^2\theta - \nu a^2\sin^2\theta} \tag{4.10}$$

$$\begin{aligned}\hat{\omega} = &\frac{aQ}{\sqrt{2}}\frac{(1+2\nu)r^2+\nu a^2\sin^2\theta+(\nu+1)(1+4\nu)a^2\cos^2\theta}{(r^2+(\nu+1)a^2\cos^2\theta-\nu a^2\sin^2\theta)^2}\sin^2\theta d\phi + \\ &+\frac{aQ}{\sqrt{2}}\frac{(1+2\nu)r^2+(\nu+1)a^2\cos^2\theta-\nu(3+4\nu)a^2\sin^2\theta}{(r^2+(\nu+1)a^2\cos^2\theta-\nu a^2\sin^2\theta)^2}\cos^2\theta d\psi\,,\end{aligned} \tag{4.11}$$

as well as the metric of the base space:

$$\begin{aligned}h_{mn}dx^m dx^n = &f\left[\frac{dr^2}{r^2+a^2}+d\theta^2\right] - \frac{a^4}{2f}\nu(\nu+1)\sin^2 2\theta d\phi d\psi + \\ &+\frac{1}{f}\left\{(r^2+a^2\cos^2\theta)(r^2+a^2)+\nu a^2\cos^2\theta(2r^2+(\nu+2)a^2)\right\}\sin^2\theta d\phi^2 + \\ &+\frac{1}{f}\left\{(r^2+a^2\cos^2\theta)r^2+\nu a^2\sin^2\theta(-2r^2+\nu a^2)\right\}\cos^2\theta d\psi^2\,.\end{aligned} \tag{4.12}$$

To simplify the expressions we introduced a convenient notation:

$$f \equiv r^2+(\nu+1)a^2\cos^2\theta - \nu a^2\sin^2\theta,. \tag{4.13}$$

We also traded the radius of y direction for the parameter a:

$$a = \frac{Q}{R} \tag{4.14}$$

and from now on the radius R will not appear in the solution. Jumping ahead, we just mention that the relation (4.14) holds for the near horizon solution, but it will look slightly different for the one which is asymptotically flat.

In equations (4.9) and (4.11) we also put the hats over H and ω to stress the fact that we are dealing with near horizon region.

Now we are ready to perform the final step of our program: to go from the space with $AdS_3 \times S^3$ asymptotics to the asymptotically flat space by performing the map (3.6). Let us introduce

$$\tilde{\omega} = \omega - \hat{\omega} \tag{4.15}$$

[13]Another reason to concentrate on integer ν comes from CFT: since we want to flow from one state in the Ramond sector to another state in the same sector, then ν has to be an integer. The detailed discussion of relation between spectral flow and supergravity solutions is beyond the scope of this paper.

Then (3.6) gives an equation

$$(d\tilde{\omega})^+ = \frac{1}{\hat{H}}(d\hat{\omega})^+ + \frac{1}{2}\frac{F}{\hat{H}}d\beta\,. \tag{4.16}$$

This equation can be solved by taking

$$\tilde{\omega} = \sqrt{2}\nu(\nu+1)a^3 \frac{\sin^2\theta d\phi - \cos^2\theta d\psi}{r^2 + (\nu+1)a^2\cos^2\theta - \nu a^2\sin^2\theta} = \frac{2\nu(\nu+1)a^2}{Q}\beta\,. \tag{4.17}$$

To summarize, we found an asymptotically flat solution of type IIB supergravity which has a form

$$\begin{aligned} ds^2 &= -2H^{-1}(du+\beta_m dx^m)\left(dv + \omega_m dx^m + \frac{F}{2}(du+\beta_m dx^m)\right) + \\ &\quad + H h_{mn} dx^m dx^m + \sum_{i=1}^{4} dz_i dz_i \\ G &= \frac{1}{2}\,{}^\star dH - \frac{1}{2}H^{-1}(du+\beta)\wedge(d\omega)^- + \\ &\quad + \frac{1}{2}H^{-1}\left[dv + \omega + \frac{F}{2}(du+\beta)\right]\wedge[d\beta + (du+\beta)\wedge dH] \end{aligned}$$

with coefficients given by (4.9)–(4.12) and

$$H = 1 + \hat{H}\,, \qquad \omega = \hat{\omega} + \frac{2\nu(\nu+1)a^2}{Q}\beta\,. \tag{4.18}$$

We claim that this solution is completely regular and it carries three nontrivial charges: D1, D5 and momentum.[14]

5. Properties of the solution

In this section we will analyze the solution in more detail and we will show that it possesses all the right properties to be one of the microscopic states which contribute to the entropy of the three charged black hole.

1. The solution is asymptotically flat.

2. The solution does not have curvature singularities. A sufficient condition for the absence of curvature singularities is to have a well-defined metric and inverse metric. A potential problem with metric $g_{\mu\nu}$ may only arise when one of the coefficient functions blows up. This could happen only in the regions where $f = 0$ or $H = 0$. To avoid complications associated with $H = 0$, we restrict our attention to the range of parameters in which H never vanishes:[15]

$$\nu > 0 \;\;\Rightarrow\;\; a^2 < \frac{Q}{\nu}, \qquad \text{or} \qquad \nu < 0 \;\;\Rightarrow\;\; a^2 < \frac{Q}{|\nu+1|}\,. \tag{5.1}$$

[14] Notice that taking $\nu = 0$ we recover the asymptotically flat solution of [14, 15] in the form it was presented in [23]. Of course, the case $\nu = 0$ is special, since the solution has only two charges (D1 and D5).

[15] This does not mean however that H has a definite sign. But the regions with positive and negative H are connected through $H = \infty$, not $H = 0$.

While it would be interesting to understand the properties of the solution outside the range (5.1), our goal is to present the simplest example of the regular solution, so we will assume that (5.1) is satisfied. Under this assumption, particular components of the metric $g_{\mu\nu}$ can only diverge if $f = 0$.

In the regions where the metric $g_{\mu\nu}$ is regular, the inverse metric is well-defined as long as $g_{\mu\nu}$ has non-vanishing determinant. By direct computation one can check that

$$\det\ g = -\frac{1}{4} H^2 r^2 \sin^2 2\theta f^2 \,, \tag{5.2}$$

so the inverse metric and curvature invariants can only have problems if at least one of the conditions

$$r = 0 \,, \qquad \theta = 0 \,, \qquad \theta = \frac{\pi}{2} \,, \qquad f = 0 \tag{5.3}$$

is satisfied. Let us analyze these regions one by one.

(a) We first show that points where $\theta = 0$ or $\theta = \frac{\pi}{2}$ correspond to coordinate singularities. To see this we go from spherical coordinates (r, θ, ψ, ϕ) to cartesian coordinates in the standard way:

$$\begin{aligned} x_1 &= r \sin\theta \cos\phi \,, \qquad & x_2 &= r \sin\theta \sin\phi \\ x_3 &= r \cos\theta \cos\psi \,, \qquad & x_2 &= r \cos\theta \sin\psi \,. \end{aligned} \tag{5.4}$$

Then various components of the metric may acquire additional singularities as r goes to zero, but not at the points where $f \neq 0$ and $r \neq 0$. At such points we can compute the determinant of the new metric by multiplying (5.2) and appropriate jacobian:

$$\det\ g' = -\frac{1}{r^4} H^2 f^2 \,. \tag{5.5}$$

This demonstrates that in Cartesian coordinates both $g_{\mu\nu}$ and inverse metric are well-defined away from the points where $f = 0$ or $r = 0$, so the singularities at $\theta = 0, \frac{\pi}{2}$ are the usual coordinate singularities of the spherical frame.

(b) We now look at the vicinity of the region where $r = 0$. One can write an approximate expression for the metric of the base space:

$$\begin{aligned} h_{mn} dx^m dx^n \approx\ & f \left[\frac{dr^2}{a^2} + d\theta^2 \right] + \frac{a^2}{f} (a^2 \sin^2\theta \cos^2\theta + r^2) [(\nu + 1) d\phi - \nu d\psi]^2 - \\ & - \frac{a^2}{f} \left(\cos 2\theta + \frac{1 + 2\nu}{4} (3 + \cos 4\theta) \right) r^2 [(\nu + 1) d\phi - \nu d\psi][d\phi - d\psi] + \\ & + \frac{a^2}{f} ((1 + \nu) \cos^2\theta - \nu \sin^2\theta)^2 r^2 [d\phi - d\psi]^2 \,. \end{aligned} \tag{5.6}$$

An explicit form of this metric is not important for us, what is important is that this metric is regular in the new coordinate system:

$$\tilde{\phi} = (\nu + 1)\phi - \nu\psi \,, \qquad \tilde{\psi} = \psi - \phi \,, \qquad x_1 = r \cos\tilde{\psi}, \quad x_2 = r \sin\tilde{\psi} \,. \tag{5.7}$$

In this coordinate system the determinant (5.2) gets multiplied by the appropriate jacobian to become

$$\det\ g' \sim H^2 \sin^2 2\theta f^2 \,, \tag{5.8}$$

so both direct and inverse metrics are regular at $r = 0$ unless $f = 0$ as well. To be more precise, the introduction of cartesian coordinates may lead to new singularities in metric components if $d\tilde{\psi}$ appears without r^2 in front of it. However, one can check that such singularities can be eliminated by an additional diffeomorphism

$$u \to \tilde{u} = u - \frac{Q}{\sqrt{2}a}\tilde{\psi}\,, \qquad v \to \tilde{v} = v + \frac{Q}{\sqrt{2}a}\left[1 - 2\nu(\nu+1)\frac{a^2}{Q}\right]\tilde{\psi}\,, \tag{5.9}$$

We present the details in the appendix B. This completes the proof of regularity at $r = 0$ and generic value of θ.

We still have to show that metrics are regular if both $r = 0$ and $\theta = 0$ (or $r = 0$ and $\theta = \pi/2$). One can check that all curvature invariants stay finite as we approach such points (we present some details in the appendix B). However for generic values of the parameters, the solution develops a conical singularity at $r = 0$ and poles on the sphere. The singularity is absent if and only if the parameters satisfy the relations (B.12):[16]

$$R = \frac{Q}{a}|m-1|, \qquad \nu(\nu+1) = \frac{Qm}{a^2} \tag{5.10}$$

with some integer m. Notice that for $m = 0$ this reduces to the familiar regularity condition for D1-D5 (4.4):

$$R = \frac{Q}{a} \tag{5.11}$$

We also notice that relation (5.10) is different from the condition which we had in the near horizon region:

$$R = \frac{Q}{a}, \qquad \nu = m \tag{5.12}$$

and generically ν is (5.10) is not an integer.

The source of this difference is easy to explain: the near horizon limit of asymptotically flat solution is supposed to work only in the vicinity of the surface $f = 0$. However for $m \neq 0$ the points $r = 0, \theta = 0$ are not close to this surface, so it is not surprising that the regularity condition was modified.

(c) Finally we consider a vicinity of the points where function f vanishes. The analysis is somewhat technical, and we refer to the appendix B for the details. Here we just state that the solution does not have singularities at such points.

To summarize, we have shown that all apparent singularities of the solution (4.9)–(4.12), (4.18), (4.18) can be traced to a bad choice of coordinates, and all curvature invariants are well-defined and finite for our solution. This means that the solution is completely regular.

3. The solution does not have a horizon. To check this claim one needs to study the global properties of geodesics, and we will not present such detailed analysis here. Instead we will look for one of the *symptoms* of the horizon: infinite redshift of the frequency. This will allow us to rely on *local* analysis, and although it would not rigorously prove the absence of the horizon, it will give a strong argument in support of this claim.

[16]From the point of view of an observer at infinity the first of the relations (5.10) should be interpreted backward. One can fix the charge of the solution Q, the radius of y direction R and the integer m, then (5.10) determines a and ν. Notice that ν is not necessarily an integer.

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So we want to look for the surfaces of infinite redshift, i.e. surfaces where g^{tt} blows up. As we already observed in the analysis of possible singularities, the components of the inverse metric (including g^{tt}) can only blow up at the points (5.3) where either individual components of the metric diverge or determinant of the metric vanishes. Now we have to go through the same list.

(a) For $\theta = 0$ or $\theta = \frac{\pi}{2}$ we again go to the cartesian coordinates to see that g^{tt} stays finite.

(b) In the vicinity of the region $r = 0$ the coefficient g^{tt} does blow up, since the determinant of the metric vanishes as r^2, while the cofactor of g^{tt} behaves as

$$A^{tt} \sim -\frac{a^4\nu^2(\nu+1)^2}{4Q}\frac{Q+(\nu+1)a^2\cos^2\theta - \nu a^2\sin^2\theta}{(\nu+1)\cos^2\theta - n\sin^2\theta}\sin^2 2\theta\,. \tag{5.13}$$

However this surface of infinite redshift should not be interpreted as a horizon. The reason is that unlike the case of Schwarzschild black hole where coordinates (r, t) break down at the horizon and one needs to use analytic continuation in both of them, here at the surface $r = 0$ r is still a good coordinate. Moreover, in the coordinate frame (5.7) which regularizes this point, r is a radial coordinate, so the space is complete and we cannot continue beyond $r = 0$. This means that while the surface of infinite redshift at $r = 0$ is a candidate for a horizon, there would be no space "behind it," so we this surface should not be viewed as a horizon.[17]

(c) In the vicinity of the points where $f = 0$ the function g^{tt} goes to a finite limit. To see this we observe that near these points the metric has a form (B.21) with $\alpha = 1$ and all its components are regular. We also know that (5.2) goes to a finite limit as we approach $f = 0$, so all components of $g^{\mu\nu}$ (and in particular g^{tt}) stay finite.

To summarize, we have analyzed the surfaces of infinite redshift and we have shown that there is only one such surface ($r = 0$), but it should not be viewed as a horizon. It would be interesting to perform a more detailed study of causal structure of the space-time which we are considering to reach a definitive conclusion about the presence of the horizon. Such investigation should also shed some light on the closed time-like curves in the geometry. Here we will rely only on the local analysis, and we take the absence of the surfaces with infinite redshift as a strong indication for the absence of the horizon.

4. The solution has three charges. In order to compute them we have to look at the fall-off of various fields at infinity. The numbers of D1 and D5 branes and momentum excitations are[18] [25]:

$$n_1 = 2\frac{V}{4\pi^2 g}\int_{S^3} \star_6 G = \frac{QV}{g}\,, \quad n_5 = \frac{2}{4\pi^2 g}\int_{S^3} G = \frac{Q}{g}\,, \quad n_P = \frac{R^2V}{g^2}\nu(\nu+1) = \frac{R^2V}{g^2}\frac{Q}{a^2}m\,. \tag{5.14}$$

[17]This should be contrasted with the case of conventional three charge black hole [24] with harmonic functions $H_i = Q_i/r^2$. In that case the horizon was also located at $r = 0$, but the coordinate system was singular there, so one needs to do an analytic continuation and one indeed sees a horizon.

[18]An extra factor of two in these expressions appears due to non-traditional normalization of G in [12]

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For completeness we also present the expressions for the angular momenta:

$$J_\phi = \frac{aR}{Q}\frac{Q^2V}{g^2}\left(1+\nu\left[1+\frac{a^2}{Q}(\nu+1)\right]\right), \qquad J_\psi = \frac{aR}{Q}\frac{Q^2V}{g^2}\nu\left[1-\frac{a^2}{Q}(\nu+1)\right]. \quad (5.15)$$

5. The solution is supersymmetric. This was a starting assumption of the GMR construction [12], and our metric solves their equations.

6. Interpretation in terms of branes. [19] It is interesting to find a configuration of branes which produces the solution which we constructed. According to the usual correspondence between branes and supergravity solutions [26], at weak string coupling one starts from branes in flat space, then as g gets larger, the branes start to modify the geometry producing a nontrivial gravitational background. In this paper we constructed an example of such background and we showed that it is regular. Now we want to go to a weak string coupling and identify the corresponding configuration of branes. Since we want to keep charges fixed, we would be interested in the following rescaling:

$$g \to \epsilon g, \qquad Q \to \epsilon Q, \qquad R \to \epsilon R \quad (5.16)$$

and we keep the values of V, a, m fixed. As ϵ goes to zero, we observe that

$$\nu \sim \frac{Qm}{a^2} \quad (5.17)$$

becomes small, and then it is convenient to write function f in the form

$$f = r^2 + a^2\cos^2\theta + \nu a^2\cos 2\theta\,. \quad (5.18)$$

First we consider the region where $f \gg \nu a^2 \sim mQ$, and we always assume that $m \neq 0$. Then the metric of the base space (4.12) reduces to

$$h_{mn}dx^m dx^n = (r^2+a^2\cos^2\theta)\left[\frac{dr^2}{r^2+a^2}+d\theta^2\right] + (r^2+a^2)\sin^2\theta d\phi^2 + r^2\cos^2\theta d\psi^2\,. \quad (5.19)$$

This is just a metric of a flat four dimensional space written in the unusual coordinates (see [23] for details). One can also see that the functions F, β, ω can be dropped from (4.18) in this region and H can be replaced by one. So in the region where $f \gg mQ$ we have a usual flat space. However this condition itself cuts some region out of the flat space, and we will now describe this region. In a four dimensional space parameterized by the cartesian coordinates x_1, x_2, x_3, x_4, we take a 1–2 plane and draw a circle of radius a with a center in the origin. Then we consider a three dimensional torus which surrounds this circle and which has radii (a, d, d) where $a \gg d \gg \sqrt{mQ}$. Notice that pictorially this torus looks like a thin tube surrounding the circle. One can see that outside of this torus the condition $f \gg mQ$ is satisfied and the space is flat.

Now we look at the interior of the torus. Before we do this for the three charged system, it is useful to recall the picture for the two charges. In that case the space was

[19] I want to thank Juan Maldacena for suggesting to add this discussion.

also flat outside the torus (a, d', d') (where $a \gg d' \gg \sqrt{Q}$), and the branes themselves were located precisely on the circle $x_1^2 + x_2^2 = a^2, x_3 = x_4 = 0$. Inside the torus the system had some curved geometry, but as one approached the circle one saw a metric of the KK monopole with D1 and D5 fluxes [6]. In the three charged system we would also have some complicated geometry as we go inside the torus (a, d, d), but we want to see how this geometry ends. To analyze this one needs to consider the limit $f \ll Q$, which selects the following region in the r, θ space:

$$\tilde{\theta} \equiv \frac{\pi}{2} - \theta \sim \frac{\sqrt{Q}}{a} \ll 1, \qquad r^2 + a^2\tilde{\theta}^2 \approx a^2\nu \approx mQ\,. \tag{5.20}$$

We see that unlike the case of D1-D5 system where KK monopole looked like a point in the $(r, \tilde{\theta})$ plane, we now have a circle in this plane.[20] Of course, the $(r, \tilde{\theta})$ plane does not have a clear geometrical meaning since these are not the cartesian coordinates, so to analyze the shape of the "singularity" we have to consider the full six dimensional space. However the observation that we are dealing with a circle in $(r, \tilde{\theta})$ plane is still useful, because we see that at a generic point of this circle (when $r \neq 0$ and $\tilde{\theta} \neq 0$) the "singularity" extends along one coordinate (an angle along the circle) instead of two (r and $\tilde{\theta}$), and the other coordinate becomes transverse to the "singularity". On the other hand, at a generic point of this circle, all components of six dimensional metric are well-defined (see (B.21)) and its determinant (5.2) does not vanish, so we have a non-degenerate six dimensional space. This shows that the worldvolume of the "singularity" is $1+4$ dimensional (ignoring the four directions on the torus), as opposed to the D1-D5 system, where the corresponding worldvolume had $1+1$ dimensions. Of course we don't actually have a singularity in either case, and it would be interesting to get an intuitive understanding of the regularization mechanism for the three charged system. It is also interesting to note that in the case of D1-D5 system the space ended on the singularity, while in the case of the three charges the space continues inside the "domain wall" and ends there smoothly.

6. Discussion.

We have constructed a regular solution of type IIB supergravity which has three charges corresponding to D1, D5 branes and momentum. We showed that this solution satisfies all requirements one wants to impose on a microscopic state, so we conjecture that our solution would indeed be one of the states contributing to the entropy of three charged black hole.[21] It would be very interesting to find geometries which correspond to other microscopic states.

In the case of D1-D5 system the guiding principle which led to construction of regular geometries in [5] was based on microscopic understanding of D1-D5 bound states as being dual to a fundamental string vibrating with different profiles. It seems that we are still missing this detailed understanding of bound states with three charges, although a

[20]To be precise, we have a quarter of the circle since both r and $\tilde{\theta}$ are positive.

[21]Our solutions would contribute to the entropy of either BMPV black hole [3] or more general rotating black holes such as ones discussed in [27].

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significant progress in this direction has been made recently [28]. These papers tried to understand the three charged system in a dual frame where they describe supertube [29, 30]. Although the analysis of [28] was done on the level of worldvolume theory, one may hope that ultimately there would be some way of finding the gravitational solutions corresponding to supertubes with three charges, just like the solutions for supertubes [29] were found in [31].

Another interesting direction is to use the technology of [12] in order to study the AdS/CFT correspondence. Again an analogy with pure D1-D5 system might be useful. So far we were always talking about solutions which were asymptotically flat. However (2.11) can also be viewed as solutions with $AdS_3 \times S^3$ asymptotics. As such they would describe the geometries which are dual to Ramond vacua of the CFT, i.e. to the states satisfying

$$L_0 = \bar{L}_0 = \frac{c}{24}\,. \tag{6.1}$$

The map between such Ramond vacua and geometries was presented in [5, 6]. The momentum charge from the point of view of CFT corresponds to $L_0 - \bar{L}_0$, so in order to construct a state with nonzero momentum we clearly have to give up the relation (6.1). On the other hand, we are still interested in supersymmetric configuration, so we would like to stay in the Ramond vacuum at least in one of the sectors. Thus the geometries with nonzero momentum would correspond to the states with

$$L_0 \neq \bar{L}_0 = \frac{c}{24}\,. \tag{6.2}$$

The near horizon region of the geometry which we discussed in this paper corresponded to such state, but it was not very interesting from the AdS/CFT point of view since it was obtained form the Ramond vacuum by a spectral flow. It would be very interesting to use the technology of GRM [12] to construct the geometries corresponding to less trivial states in the CFT.

To conclude, we consider this paper as one of the first steps in understanding regular solutions with three charges, and getting more general solutions would be very important for black hole physics and for AdS/CFT.

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A. Definitions of the near horizon limit.

In this paper we reserved the term "near horizon region" for the solutions parameterized by $(\hat{H}, F, \beta, \hat{\omega}, h_{mn})$. Such solutions were defined by (3.5) which did not involve any limit. In this appendix we will show that in some circumstances the map (3.5) can indeed be interpreted as taking a limit which is analogous to the near horizon limit for the black holes [16]. Hopefully this would give a reasonable justification for the term "near horizon region."

We begin with the simple case of two charge solution (2.11), (2.9). Let us pick some point $\xi_0 \in (0, L)$ and zoom in on the region near $\mathbf{x} = \mathbf{F}(\xi_0)$. Without the loss of generality we assume that $\xi_0 = 0$, $\mathbf{F}(0) = 0$, then the "zooming in" corresponds to rescaling:

$$\mathbf{x} \to \epsilon\mathbf{x}\,, \qquad \mathbf{F}(\xi) \to \epsilon\mathbf{F}(\tilde{\xi}) \equiv \epsilon\mathbf{F}\left(\frac{\xi}{\epsilon}\right). \tag{A.1}$$

Notice that we had to rescale the argument of $\mathbf{F}$ since we want to preserve the condition (2.8). In the rescaled coordinates we find:

$$\begin{aligned} H' &= 1 + \frac{Q}{\epsilon^2 \tilde{L}} \int_{-\tilde{L}/2}^{\tilde{L}/2} \frac{d\tilde{\xi}}{[\mathbf{x} - \mathbf{F}(\tilde{\xi})]^2} \equiv 1 + \frac{1}{\epsilon^2}\hat{H}\,, \\ A'_i &= -\frac{Q}{\epsilon^2 L} \int_{-\tilde{L}/2}^{\tilde{L}/2} \frac{\dot{F}_i(\tilde{\xi}) d\tilde{\xi}}{[\mathbf{x} - \mathbf{F}(\tilde{\xi})]^2} = \frac{1}{\epsilon^2} A_i\,. \end{aligned} \tag{A.2}$$

In this appendix we use primes to denote expressions after rescaling. Taking Hodge dual with respect to primed variables, we find:

$$B'_i = \frac{1}{\epsilon^2} B_i\,. \tag{A.3}$$

Writing the metric (2.11) for primed variables we get:

$$ds^2 = (\epsilon^2 + \hat{H})^{-1} \left[-(dt - A_m dx^m)^2 + (dy + B_m dx^m)^2 \right] + (\epsilon^2 + \hat{H})\delta_{mn} dx^m dx^n + d\mathbf{z}d\mathbf{z}\,.$$

Then in the limit $\epsilon \to 0$ we recover precisely the near horizon limit parameterized by $(\hat{H}, F, \beta, \hat{\omega}, h_{mn})$.

Of course, for the profile with $\mathbf{F} = const$ this definition of near horizon limit reduces to the standard decoupling limit of [16], where the role of ϵ was played by α'. For this type of profile one can define a region which has exactly the same singularity, but it asymptotes to $AdS_3 \times S^3$.

For the profiles with non-constant $\mathbf{F}$ one should be a little more careful since there is one additional length scale in the problem (the characteristic size of $|\mathbf{F}(\xi_1) - \mathbf{F}(\xi_2)|$) which is an input parameter and which cannot be rescaled. So after taking ϵ to zero we zoom in on a vicinity of some point on the curve $\mathbf{x} = \mathbf{F}(\xi)$, but generically a part of the curve would go outside this region. So generically there would not be a good limit which preserves the entire curve and has $AdS_3 \times S^3$ asymptotics, but the map

$$H = 1 + \hat{H} \to \hat{H} \tag{A.4}$$

can still be considered as a solution generating technique which produces a geometry with such properties. Using analogy with $\mathbf{F} = const$ case we will refer to this map as a "going to the near horizon regime", but one should keep in mind that it can't always be defined a limit.

Now let us look at a more interesting case of solution with three charges (2.3) and try to justify the map (3.5) by taking some limit. From our experience with D1-D5 case we already know that this limit would involve "zooming in" on a vicinity of a point where sources of H are located. Without the loss of generality we take this point to be $\mathbf{x} = 0$. For all solutions which we consider in this paper (and we believe this would be generically true for the solutions corresponding to microscopic states), the other functions (β, ω, F) have sources at the same points as H. We observe that the system (2.4) is invariant under the following rescaling:

$$x \to x' = \epsilon x, \quad v \to v' = \frac{v}{\epsilon^2}, \qquad H \to H' = 1 + \frac{\hat{H}}{\epsilon^2}, \quad \omega \to \omega' = \frac{\omega}{\epsilon^2}, \quad F \to F' = \frac{F}{\epsilon^2} \tag{A.5}$$

while u, β and h_{mn} are kept fixed. So starting from any asymptotically flat solution and a point where H blows up, we can construct a set of solutions which is parameterized by ϵ (so that the original solution is recovered at $\epsilon = 1$) which "magnifies" a vicinity of this point. In the limit when $\epsilon \to 0$ we get the "near horizon limit" which is described by the metric

$$ds^2 = -2\hat{H}^{-1}(du + \beta_m dx^m)\left(dv + \hat{\omega}_m dx^m + \frac{F}{2}(du + \beta_m dx^m)\right) + \hat{H} h_{mn} dx^m dx^m \tag{A.6}$$

with $\hat{\omega}$ given by a solution of (3.5). To see how the replacement $\omega \to \hat{\omega}$ arises, we look at

$$dv' + \omega' = dv' + \hat{\omega}' + (\omega' - \hat{\omega}') . \tag{A.7}$$

We now recall that by definition (3.5), the form $\omega' - \hat{\omega}'$ satisfies the equation

$$[d(\omega' - \hat{\omega}')]^+ = \frac{1}{H'}(d\omega')^+ + \frac{1}{2}\frac{F'}{H'}d\beta' \sim \frac{1}{H}(d\omega)^+ + \frac{1}{2}\frac{F}{H}d\beta \,, \tag{A.8}$$

where we used $\sim$ to denote the leading order at small ϵ. Introducing $\tilde{\omega}$ as a solution of the equation

$$(d\tilde{\omega})^+ = \frac{1}{H}(d\omega)^+ + \frac{1}{2}\frac{F}{H}d\beta \tag{A.9}$$

we can rewrite (A.7) as

$$dv' + \omega' = \frac{1}{\epsilon^2}\left[dv + \hat{\omega} + \epsilon^2\tilde{\omega} + \cdots\right] . \tag{A.10}$$

So indeed in the limit $\epsilon \to 0$, ω is replaced by $\hat{\omega}$.

To summarize, we have shown that in the three charged system there exists a limit (A.5) which reproduces the "near horizon map" (3.5). However, we want to stress again that this limit only magnifies a vicinity of a particular point where H has singularity, while solution with $\hat{H}$ and $\hat{\omega}$ can be extended beyond such vicinity. In this sense, one can view the map (3.5) as a solution generating technique which was inspired by the near horizon limit.

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B. Elimination of coordinate singularities.

In this appendix we will provide some extra details on how the apparent singularities of the solution (4.9)–(4.12), (4.18), (4.18) can be eliminated by an appropriate change of coordinates. While the basic ideas of such reparameterizations were outlined in section 5, there we skipped some technical details which will be explained in this appendix.

This appendix consists of two main parts: one deals with singularities at $r = 0$, but $f \neq 0$, and another deals with vicinity of regions where $f = 0$ without any limitation on the value of r. This is a very natural split, since $r = 0$ was a singularity associated with our choice of spherical coordinates, while $f = 0$ seemed to be a real singularity where the harmonic functions had their sources. We will show that contrary to a naive expectation, the solution is completely regular at those points as well.

1. Singularity at $r = 0$. We already did a partial analysis of this singularity in section 5, here we will present some details we skipped before, and we also analyze the poles on the sphere ($\theta = 0$ of $\theta = \pi/2$) when they located at $r = 0$.

We begin with generic case ($\theta \neq 0$, $\theta \neq \pi/2$). As we already mentioned in section 5, the metric of the base space(5.6) becomes regular after the change of variables[22] (5.7)

$$\tilde{\phi} = (\nu+1)\phi - \nu\psi\,, \qquad \tilde{\psi} = \psi - \phi\,, \qquad x_1 = r\cos\tilde{\psi}\,, \qquad x_2 = r\sin\tilde{\psi}\,. \tag{B.1}$$

Here we will show that this change of variables does not introduce any new singularities into $g_{\mu\nu}$. We already know that there are no singularities in h_{mn}, so we have to analyze

$$du + \beta \approx du + \frac{Q}{a\sqrt{2}}\left[-d\tilde{\psi} + \frac{\cos 2\theta}{(\nu+1)\cos^2\theta - \nu\sin^2\theta}d\tilde{\phi}\right] \tag{B.2}$$

and

$$\begin{aligned} dv + \omega \approx\ & dv + \frac{Q}{a\sqrt{2}}\left[d\tilde{\psi} + \frac{(\nu + \cos^2\theta)d\tilde{\phi}}{[(\nu+1)\cos^2\theta - \nu\sin^2\theta]^2}\right] + \\ & + \sqrt{2}a\nu(\nu+1)\left[-d\tilde{\psi} + \frac{\cos 2\theta}{(\nu+1)\cos^2\theta - \nu\sin^2\theta}d\tilde{\phi}\right]. \end{aligned} \tag{B.3}$$

To have regular metric we have to avoid an appearance of $d\tilde{\psi}$ unless it is multiplied by r. Clearly this can be achieved by diffeomorphism (5.9). This concludes our analysis of the singularity at $r = 0$ and generic values of θ, we now proceed with analysis of the poles on the sphere.

First we look at $r = 0$, $\theta = 0$. Near this point the metric (5.6) simplifies:

$$h_{mn}dx^m dx^n \approx (\nu+1)(dr^2 + a^2 d\theta^2) + \frac{a^2\theta^2}{\nu+1}\left[(\nu+1)d\phi - \nu d\psi\right]^2 + \frac{r^2}{\nu+1}d\psi^2 \tag{B.4}$$

For the relevant one-forms we get:

$$\begin{aligned} du + \beta &\approx du + \frac{Q}{a\sqrt{2}}\left[-\frac{d\psi}{\nu+1} + \frac{\theta^2 d\phi}{\nu+1}\right], \\ dv + \omega &\approx dv + \frac{Q}{a\sqrt{2}}\left[\frac{1+4\nu}{\nu+1}\theta^2 d\phi + \frac{d\psi}{\nu+1}\right] + a\sqrt{2}\nu\left[-d\psi + \theta^2 d\phi\right] \end{aligned} \tag{B.5}$$

[22]We will also need the inverse relation between angles: $\phi = \tilde{\phi} + \nu\tilde{\psi}$, $\psi = \tilde{\phi} + (\nu+1)\tilde{\psi}$.

and the scalar functions are

$$H = 1 + \frac{Q}{a^2(\nu+1)}, \qquad F = -2\nu\,. \tag{B.6}$$

Substituting this in (4.18) we get an approximate metric near the points where $r = 0, \theta = 0$:

$$\begin{aligned} ds^2 \,=& -\frac{[Q+(\nu+1)a^2][Q-\nu(\nu+1)a^2]}{(\nu+1)a^2} d\tilde{U} d\tilde{V} + \\ &+\frac{(\nu+1)Q}{Q+(\nu+1)a^2}(dr^2+a^2d\theta^2) + \frac{(\nu+1)a^2Qr^2}{[Q+(\nu+1)a^2][Q-\nu(\nu+1)a^2]^2}dy^2 + \\ &+\frac{(\nu+1)Qa^2\theta^2}{Q+(\nu+1)a^2}\left\{d\phi + \frac{\nu(\nu+1)a^3}{Q[Q-\nu(\nu+1)a^2]}dy\right\}^2 + \\ &+r^2dy^2(A_1 d\tilde{U} + A_2 d\tilde{V}) + \theta^2\left\{d\phi + \frac{\nu(\nu+1)a^3}{Q[Q-\nu(\nu+1)a^2]}dy\right\}(A_3 d\tilde{U} + A_4 d\tilde{V})\,. \end{aligned} \tag{B.7}$$

Here we introduced $\tilde{U}$ and $\tilde{V}$ as some special linear combinations of t, y, ϕ, ψ with constant coefficients. Neither explicit form these combinations nor the expressions for the constants A_1, A_2, A_3, A_4 are important for our discussion, so we do not write them down to avoid unnecessary complications.

The expression (B.7) clearly demonstrates that all curvature invariants stay finite as we *approach* the point $r = 0, \theta = 0$. However we still can encounter a conical singularity located exactly at that point. To see this let us look at the following two terms:

$$\begin{aligned} &\frac{(\nu+1)Q}{Q+(\nu+1)a^2}dr^2 + \frac{(\nu+1)a^2Qr^2}{[Q+(\nu+1)a^2][Q-\nu(\nu+1)a^2]^2}dy^2 = \\ &\quad = \frac{(\nu+1)Q}{Q+(\nu+1)a^2}\left\{dr^2 + r^2\left(\frac{a\,dy}{Q-\nu(\nu+1)a^2}\right)^2\right\}. \end{aligned} \tag{B.8}$$

This metric has a conical singularity unless we make an identification $y \sim y + 2\pi R$ with

$$R = \frac{Q - \nu(\nu+1)a^2}{a}\,. \tag{B.9}$$

Suppose we make such identification (we recall that coordinate y was periodic to begin with and this identification simply chooses a particular relation between the values of R and a). Then we can trade a parameter a in (B.7) for the radius of y circle:[23]

$$\begin{aligned} ds^2 \,=& -R\frac{Q+(\nu+1)a^2}{(\nu+1)a}d\tilde{U}d\tilde{V} + \frac{(\nu+1)Q}{Q+(\nu+1)a^2}\left(dr^2 + r^2\frac{dy^2}{R^2}\right) + \\ &+\frac{(\nu+1)Qa^2}{Q+(\nu+1)a^2}\left(d\theta^2 + \theta^2\left\{d\phi + \frac{\nu(\nu+1)a^2}{Q}\frac{dy}{R}\right\}^2\right) + \\ &+r^2dy^2(A_1 d\tilde{U} + A_2 d\tilde{V}) + \theta^2\left\{d\phi + \frac{\nu(\nu+1)a^2}{Q}\frac{dy}{R}\right\}(A_3 d\tilde{U} + A_4 d\tilde{V})\,. \end{aligned} \tag{B.10}$$

[23] We still kept a in some constants in the metric (B.10). We implicitly assume that a entering those constants is expressed in terms of R by solving a quadratic equation coming from (B.9).

We see that in order to avoid a conical singularity at $\theta = 0$, the relation

$$\frac{\nu(\nu+1)a^2}{Q} = m \tag{B.11}$$

should be satisfied for some integer m.

To summarize, we found that our solution has regular curvature invariants as we approach a point $r = 0, \theta = 0$, but at that point itself the solution generically has a conical singularity. However if the parameters of the solution satisfy two relations

$$R = \frac{Q - \nu(\nu+1)a^2}{a}, \qquad a^2 = \frac{Qm}{\nu(\nu+1)} \tag{B.12}$$

for some integer m, then the solution is completely regular.

Now we look at the vicinity of a point where $r = 0, \theta = \frac{\pi}{2}$. Fortunately, we don't have to do any new analysis there since the solution has a Z_2 symmetry:

$$\theta \to \frac{\pi}{2} - \theta\,, \qquad \phi \to -\psi, \quad \psi \to -\phi\,, \qquad \nu \to -(\nu+1)\,. \tag{B.13}$$

This symmetry shows that our solution is regular at $r = 0, \theta = \frac{\pi}{2}$ as long as (B.12) is satisfied.

2. Singularity at the sources of harmonic functions. Let us now analyze the vicinity of points where the coefficient functions $(H, F, \beta, \omega, h_{mn})$ diverge. This happens in the points where $f = 0$. We will use the following trick. Instead of H and ω we introduce the following functions:

$$H_\alpha = \alpha + \frac{Q}{f}\,, \qquad \omega_\alpha = \hat{\omega} + \alpha(\omega - \hat{\omega}) = \hat{\omega} + \alpha\frac{2\nu(\nu+1)a^2}{Q}\beta\,. \tag{B.14}$$

Obviously $\alpha = 1$ corresponds to the original functions, while $\alpha = 0$ corresponds to the near horizon functions $\hat{H}$ and $\hat{\omega}$. We also notice that the terms proportional to α in H_α and ω_α become less and less relevant as one approaches the singularity (i.e. as $f \to 0$), so one would hope that very close to singularity the metric is almost the same as for the near horizon case, and the contributions with various powers of α would be suppressed. We will show that the terms containing higher powers of α can indeed be treated as small perturbation of the metric with $\alpha = 0$. Since that metric was regular by construction (see (4.7)), the perturbation does not destroy regularity, and this will serve as a proof of the desired result.

Let us now implement this strategy. We write the metric corresponding to the solution $(H_\alpha, F, \beta, \omega_\alpha, h_{mn})$ keeping only the terms α^0 and α^1:

$$\begin{aligned} ds_\alpha^2 &= -2H_\alpha^{-1}(du+\beta)\left(dv + \omega_\alpha + \frac{F}{2}(du+\beta)\right) + H_\alpha h_{mn}dx^m dx^m \\ &= ds_0^2 + 2\alpha\frac{f^2}{Q^2}(du+\beta)\left(dv + \hat{\omega} + \frac{F}{2}(du+\beta)\right) + \alpha h_{mn}dx^m dx^m - \\ &\quad - 4\alpha\nu(\nu+1)\frac{a^2 f}{Q^2}\beta(du+\beta) \\ &= \left(1 + \frac{\alpha f}{Q}\right)ds_0^2 + 4\alpha\frac{f^2}{Q^2}(du+\beta)\left(dv + \hat{\omega} + \frac{F}{2}(du+\beta) - \nu(\nu+1)\beta\frac{a^2}{f}\right). \end{aligned} \tag{B.15}$$

In this expression ds_0 denotes the metric (4.7) which was completely regular everywhere including the region which we are considering. The bracket in the first term describes a perturbation which goes to zero as we approach the "singularity" and thus it does not spoil the solution. In the second term the contributions containing du or dv go to finite limits as f goes to zero, however there is a potentially dangerous piece proportional to $1/f$:

$$4\alpha\frac{f^2}{Q^2}\beta\left(\hat{\omega}+\frac{F}{2}\beta-\nu(\nu+1)\beta\frac{a^2}{f}\right). \tag{B.16}$$

Miraculously the singular contributions in this term cancel out. To see this we rewrite $\hat{\omega}$ in the form:

$$\hat{\omega}=\frac{2a^2}{f}\nu(\nu+1)\beta+\frac{aQ}{\sqrt{2}f}(2\nu+1)(\cos^2\theta d\psi+\sin^2\theta d\phi)\,. \tag{B.17}$$

Then we find:

$$4\alpha\frac{f^2}{Q^2}\beta\left(\hat{\omega}+\frac{F}{2}\beta-\nu(\nu+1)\beta\frac{a^2}{f}\right)=4\alpha\frac{f^2}{Q^2}\beta\left\{\frac{aQ}{\sqrt{2}f}(2\nu+1)(\cos^2\theta d\psi+\sin^2\theta d\phi)\right\} \tag{B.18}$$

and we see that the singular piece indeed disappears and we have a regular perturbation of the metric near a point where $f=0$:

$$ds_\alpha^2=\left(1+\frac{\alpha f}{Q}\right)ds_0^2+\frac{4\alpha}{Q^2}(fdu+f\beta)\left(fdv+\frac{aQ}{\sqrt{2}}(2\nu+1)(\cos^2\theta d\psi+\sin^2\theta d\phi)\right).$$

We now look at the subleading orders in α. The term proportional to α^2:

$$\begin{aligned}-2\frac{\alpha^2f^3}{Q^3}(du+\beta)\left(dv+\hat{\omega}+\frac{F}{2}(du+\beta)\right)+4\nu(\nu+1)\frac{\alpha^2a^2f^2}{Q^3}(du+\beta)\beta= \\ =-2\frac{\alpha^2f^3}{Q^3}(du+\beta)\left(dv+\frac{aQ}{\sqrt{2}}(2\nu+1)(\cos^2\theta d\psi+\sin^2\theta d\phi)-\nu(\nu+1)\frac{a^2}{f}\beta\right)\end{aligned} \tag{B.19}$$

goes to finite limit as $f\to 0$, and all other terms vanish as we approach the "singularity". Indeed, the contribution of the order α^k looks like

$$(-1)^k\left[-2\frac{\alpha^kf^{k+1}}{Q^{k+1}}(du+\beta)\left(dv+\hat{\omega}+\frac{F}{2}(du+\beta)\right)+4\nu(\nu+1)\frac{\alpha^kf^k}{Q^{k+1}}(du+\beta)\beta\right] \tag{B.20}$$

so it goes to zero as f^{k-2} for $k\geq 3$.

Let us now collect the terms with all powers of α and resum the series to produce an *exact* metric:

$$\begin{aligned}ds_\alpha^2=&\left(1+\frac{\alpha f}{Q}\right)ds_0^2+\frac{4\alpha}{Q^2}(fdu+f\beta)\left(fdv+\frac{aQ}{\sqrt{2}}(2\nu+1)(\cos^2\theta d\psi+\sin^2\theta d\phi)\right)- \\ &-2\frac{\alpha^2f^3}{Q^3}\left(1+\frac{\alpha f}{Q}\right)^{-1}(du+\beta)\times \\ &\times\left(dv++\frac{aQ}{\sqrt{2}}(2\nu+1)(\cos^2\theta d\psi+\sin^2\theta d\phi)-\nu(\nu+1)\frac{a^2}{f}\beta\right)\end{aligned} \tag{B.21}$$

The series which we had to compute is a simple geometric series and it converges when f becomes small enough (i.e. if $\alpha f/Q < 1$). We can now take $\alpha = 1$ (and we can always go close enough to the surface where $f = 0$, so that the geometric series converges), then from the analysis presented above, we can see that the metric is completely regular apart from possible conical defects. The location of conical defects is dictated by the structure of ds_0^2 and (if ν is not an integer) these defects could indeed appear at either $(r = 0, \theta = 0)$ or $(r = 0, \theta = \pi/2)$. Fortunately for $\nu \neq 0$ and $\nu \neq -1$ the function f does not vanish at those points, and thus according to the analysis in the first part of this appendix, even conical defects do not arise.

It is very important that we always used the original coordinates, so the regularity of the metric $g_{\mu\nu}$ near $f = 0$ also implies the regularity of the inverse metric and thus the regularity of all curvature invariants. To see this we recall that the determinant of the metric (5.2) goes to finite limit as we approach $f = 0$, so the inverse metric is indeed regular. In particular, g^{tt} does not blow up as we approach $f = 0$, so this is not a surface of infinite redshift.

To summarize, we have shown that near any point where $f = 0$, the metric can be decomposed into a regular $AdS_3 \times S^3$ geometry ds_0^2 which comes from the near horizon limit, and smooth corrections so that the entire metric is completely regular.

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